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Mixed mode I/II brittle fracture evaluation of marble using SCB specimen

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Abstract

The failure of cracked rock masses is mainly due to the influence of both opening (mode I) and sliding (mode II) of deformation. In this paper, mixed mode I/II fracture toughness of an Iranian white marble (Harsin marble) was investigated both experimentally and theoretically. Several mixed mode fracture tests were conducted on Harsin marble using cracked semi circular bend (SCB) specimens to obtain the mixed mode fracture resistance envelope in the complete range from pure mode I to pure mode II. However, the experimental results were not consistent with the classical mixed mode brittle fracture criteria. It was demonstrated that the low fracture resistance of the tested rock was mainly due to the influence of large positive T -stresses that exist in the SCB specimen when the specimen is subjected to mixed mode loading. It was also shown that the GMTS criterion which uses three fracture parameters i.e. K_I , K_{II} and T can provide significantly better estimates for mixed mode fracture in the Harsin marble when tested by the SCB specimens.

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Nomenclature

a	Crack length
a/R	Crack length ratio
B	Specimen thickness
$GMTS$	Generalized maximum tangential stress
K_I	Mode I stress intensity factor

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K_{II}	Mode II stress intensity factor
K_{Ic}	Pure mode I fracture toughness
K_{IIc}	Pure mode II fracture toughness
MTS	Maximum tangential stress
P	Applied load
R	Radius of CCCD and SCB specimens
r, θ	Crack tip co-ordinates
r_c	Critical distance from tip crack tip
SCB	Semi circular bend specimen
$2S$	Loading span in the SCB test
S/R	Loading point span ratio in the SCB specimen
T	T -stress
T^*	Normalized T -stress
Y_I	Mode I geometry factor
Y_{II}	Mode II geometry factor
Greek symbols	
α	Crack inclination angle
θ_0	Crack initiation direction
$\sigma_{\theta\theta}$	Tangential stress component
σ_t	Tensile strength
$\sigma_{\theta\alpha}$	Critical tangential stress component

1. Introduction

Brittle fracture takes place very often in cracked components and structures made of brittle and quasi brittle materials like ceramics, some polymers and rocks. Most of the rock fractures occur under mixed mode loading because of the rock arbitrary orientation of cracks relative to the applied load or due to complex loading conditions. For example, in many applications related to rock fracture mechanics, like tunnelling, mining, rock cutting, hydraulic fracturing of gas and oil wells, stability analysis of rock slopes, etc., the failure of rock masses is mainly due to the influence of both opening (mode I) and sliding (mode II) of deformation. Therefore, it is necessary to investigate the mixed mode I/II fracture behaviour of such structures using suitable experimental and theoretical methods. The edge cracked semi circular bend (SCB) specimen under three-point bending is one of the suitable candidate specimens for conducting the fracture toughness tests on rocks and geomaterials. The major advantages in using this configuration are: convenience of test specimen preparation from the typical cylindrical rock cores, simple geometry and easy test set up with common available loading fixtures, simple manufacturing procedure without any additional machining operation, introducing full mode mixities from pure mode I to pure mode II and application of compressive load which is more suitable for rocks rather than the tensile loads. Hence this specimen has been widely used by researchers for mixed mode fracture toughness study of rocks [e.g. 1-4]. In this research, the mixed mode I/II fracture toughness of a marble rock is investigated both experimentally and theoretically using the SCB specimen. It is shown that the conventional fracture criteria that use only stress intensity factors are not able

to estimate the obtained data for the tested SCB specimens made of marble. However, an improved fracture criterion called the generalized maximum tangential stress (GMTS) criterion can provide very good estimates for the obtained test results.

2. SCB Specimen

Fig. 1 shows the geometry and loading condition of SCB specimen. The specimen is a half disc of radius R and thickness B which contains an edge crack of length a . The specimen is loaded under three-point bending with a span of $2S$ for the bottom support distance. When the crack is vertical (or is along the direction of applied load), pure mode I is achieved. By changing the angle of crack (α) relative to the applied load P , the SCB specimen experiences mixed mode I/II loading conditions. Pure mode II is achieved at a specific crack angle which depends on the ratios of a/R and S/R . Thus, various mode mixities from pure mode I to pure mode II can be achieved in the SCB specimen.

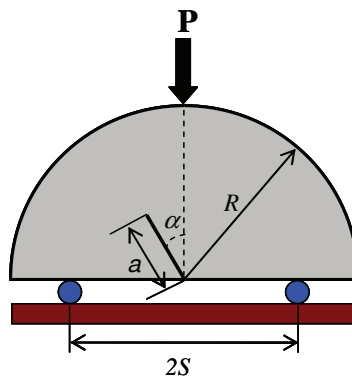


Fig. 1. Geometry and loading conditions in the SCB specimen

The mode I and mode II stress intensity factors (K_I and K_{II}) in the SCB specimen are functions of the specimen size, the crack length, the applied load, the distance between the bottom supports ($2S$) and the crack orientation angle. Thus, K_I and K_{II} are often written as:

$$K_I = Y_I \frac{P\sqrt{\pi a}}{2RB} \quad (1)$$

$$K_{II} = Y_{II} \frac{P\sqrt{\pi a}}{2RB} \quad (2)$$

where, Y_I and Y_{II} are the geometry factors corresponding to mode I and mode II. The dimensionless parameters Y_I and Y_{II} have been calculated by Ayatollahi and Aliha [5] for different geometry parameters. In the next section, the experimental procedure for conducting the fracture tests using the SCB specimens is described.

3. Fracture Toughness Experiments

For preparing the SCB specimens first a few cylinders with a diameter of 110 mm were cut from a marble block. The selected marble was from Harsin area (a region in western of Iran) and was homogenous and white in colour. Then the rock cylinders were sliced by a rotary diamond saw. The thickness of sliced discs was approximately 25 mm. Then, the disc specimens were split in two halves by using a very narrow saw blade. A thin fret saw blade of 0.5 mm thickness was used for creating the edge cracks in the SCB specimens along the required inclination angles. The ratios of a/R and S/R were chosen as 0.3 and 0.43, respectively for conducting the experiments. For this geometry and loading conditions, pure mode II angle is 50° [5]. Thus, for covering the full mode mixities, the following crack inclination angles were considered: $\alpha = \{0^\circ$ (pure mode I), 10° , 20° , 30° , 40° , 43° , 47° and 50° (pure mode II)}. For each crack angle at least three SCB samples were tested. The SCB specimens were located inside a three-point bend fixture and then loaded by a servo hydraulic compressive test machine having the capacity

of 50 kN. The speed of crosshead was set at a fixed rate of 0.5 mm/min for all the experiments. The specimens were loaded monotonically until the final fracture and the complete load–displacement data were recorded during each test. The load–displacement curves for all the samples were linear and the test samples were fractured suddenly from the crack tip. This implies that the tested rock behaved as a linear elastic material and failed in a brittle manner. Fig. 2 shows the loading setup and the fracture path for one of the broken SCB specimens.



Fig. 2. Loading setup and fracture path for one of the tested specimens

The critical stress intensity factors at the onset of fracture were determined for each tested sample using Eqs. (1) and (2) and the geometry factors given in [5]. The variations of Y_I and Y_{II} versus crack angles (α) are presented in Fig. 3 for the tested SCB specimen (with $a/R = 0.3$ and $S/R = 0.43$).

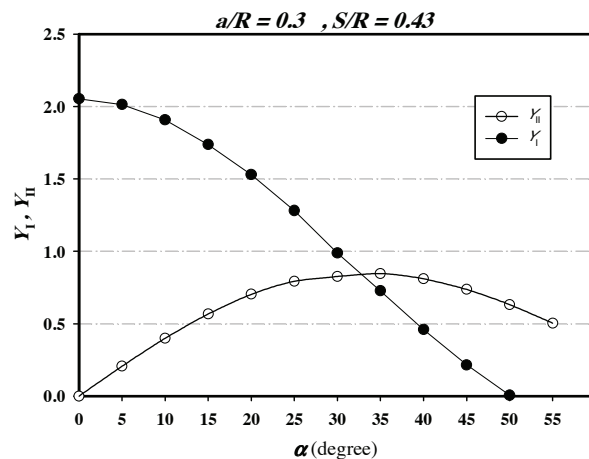


Fig. 3. Mode I and mode II geometry factors (Y_I and Y_{II}) for the tested SCB specimens (with $a/R = 0.3$ and $S/R = 0.43$)

Fig. 4 shows the test results obtained for the SCB specimens in a non-dimensional $K_{II}/K_{Ic}-K_I/K_{Ic}$ diagram. The mode I fracture toughness K_{Ic} was determined by averaging the test results corresponding to $\alpha = 0^\circ$. Also shown in this figure is the estimates of three conventional fracture criteria i.e. the maximum tangential stress (MTS) [6], the minimum strain energy density (SED) criterion [7] and the maximum energy release rate [G] criterion. According to Fig. 4, the experimentally obtained results are generally lower than the estimates of the conventional fracture criteria (like MTS criterion [6]) especially for predominantly mode II conditions. In the next section, a generalized criterion is used for better estimates for the obtained test results.

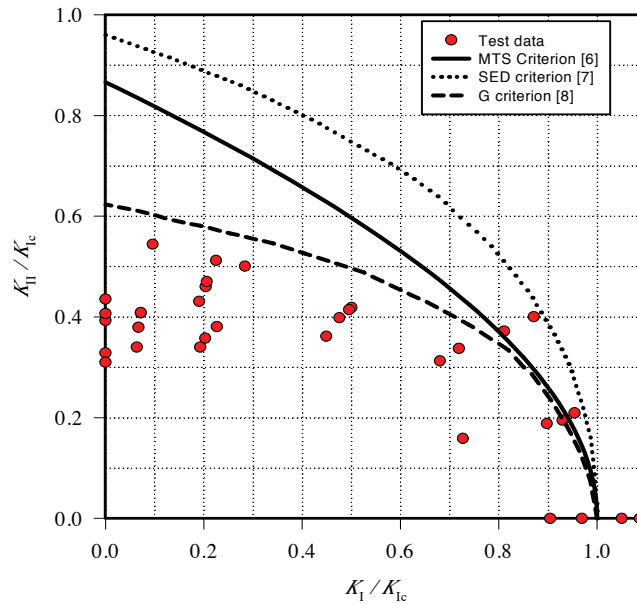


Fig. 4. Comparison of mixed mode fracture toughness results obtained for the tested marble material with estimates of three conventional fracture criteria

4. Fracture Theory

The generalized maximum tangential stress (or GMTS) criterion takes into account the influence of T -stress in addition to the mode I and mode II stress intensity factors. Hence in comparison with the conventional MTS criterion, this criterion uses a more accurate description for the tangential stress $\sigma_{\theta\theta}$ in front of the crack tip under mixed mode I/II loading as [9]:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] + T \sin^2 \theta + O(r^{1/2}) \quad (3)$$

where r and θ are the crack tip co-ordinates, T is a non-singular and constant stress term which is independent of the distance from the crack tip, usually called the T -stress. It depends on the geometry and loading conditions and its magnitude may vary in a wide range for different test specimens. $O(r^{1/2})$ represents the remaining terms of the series expansion which are negligible near the crack tip. The GMTS criterion proposes that the crack growth initiates radially from the crack tip along the direction of maximum tangential stress θ_0 . Also the crack extension takes place when the tangential stress $\sigma_{\theta\theta}$ along θ_0 and at a critical distance r_c from the crack tip attains a critical value $\sigma_{\theta\theta c}$. Both r_c and $\sigma_{\theta\theta c}$ are assumed to be material constants. According to the GMTS criterion [10], the direction of fracture initiation angle θ_0 and the onset of mixed mode I/II brittle fracture can be found from:

$$\left. \frac{\partial \sigma_{\theta\theta}}{\partial \theta} \right|_{\theta=\theta_0} = 0 \Rightarrow [K_I \sin \theta_0 + K_{II} (3 \cos \theta_0 - 1)] - \frac{16T}{3} \sqrt{2\pi r_c} \cos \theta_0 \sin \frac{\theta_0}{2} = 0 \quad (4)$$

$$K_{Ic} = \cos \frac{\theta_0}{2} \left[K_I \cos^2 \frac{\theta_0}{2} - \frac{3}{2} K_{II} \sin \theta_0 \right] + \sqrt{2\pi r_c} T \sin^2 \theta_0 \quad (5)$$

More details about how Eqs. (4) and (5) are derived can be found in Smith et al. [10]. If the effect of T in Eqs. (4) and (5) is ignored, the GMTS criterion will be identical to the conventional MTS criterion. Based on the GMTS criterion, a negative T -stress in a cracked geometry increases the mixed mode I/II fracture resistance in brittle materials and conversely a positive T -stress decreases it [10].

5. Discussion

In order to use the GMTS criterion, the value of T -stress should be known for each test specimen. The T -stress in the SCB specimen is a function of the crack length ratio (a/R), the loading span to radius ratio (S/R) and the crack inclination angle (α); and can be written as:

$$T = \frac{T^*P}{2RB} \quad (6)$$

where T^* is the non-dimensional form of T -stress. Variations of T^* with crack inclination angle in the tested SCB specimen (with $a/R = 0.3$ and $S/R = 0.43$) extracted from [5] are shown in Fig. 5. It is seen from this figure that the T -stress in the SCB specimen increases noticeably by moving from pure mode I to pure mode II and its magnitude is considerably positive especially for predominantly mode II conditions.

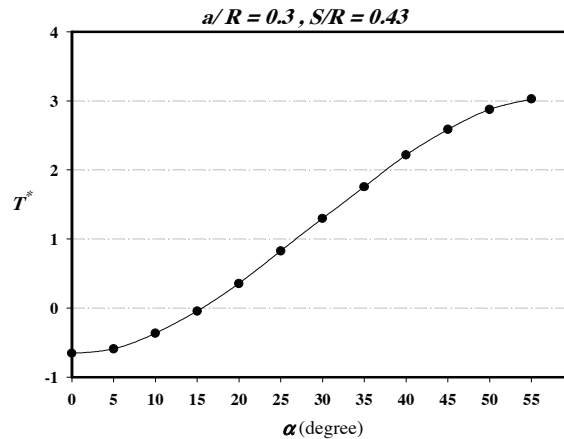


Fig. 5. Variations of normalized T -stress (T^*) in the tested SCB specimens (with $a/R = 0.3$ and $S/R = 0.43$)

In order to estimate the onset of brittle fracture using the GMTS criterion, three fracture parameters K_I , K_{II} and T should be extracted for different mode mixities from Figs. 3 and 5. Then, by substituting these parameters into Eqs. (4) and (5), the direction of fracture initiation θ_o and the onset of mixed mode fracture could be estimated by means of the GMTS criterion for any combinations of mode I and mode II in the SCB specimen.

However, for using Eqs. (4) and (5) the value of r_c (i.e. the critical distance from the crack tip) should also be known for the tested material. For rock materials, the critical distance can be related to the size of fracture process zone in front of the crack tip. This process zone is developed in rock materials due to the initiation and coalescence of micro cracks in front of the crack tip by increasing the level of applied load. According to a maximum principle stress model suggested by Schmidt [11] the size of fracture process zone in rock materials can be estimated from:

$$r_c = \frac{1}{2\pi} \left(\frac{K_{Ic}}{\sigma_t} \right)^2 \quad (7)$$

where σ_t is the rock tensile strength. By using the corresponding values of $K_{Ic} = 1 \text{ MPa}\sqrt{\text{m}}$ and $\sigma_t = 7.2 \text{ MPa}$ obtained in our experiments for the tested marble rock, the value of r_c was found as 3 mm for this material. Fig. 6 shows the results obtained from the GMTS criterion for mixed mode fracture toughness of SCB specimen made of Harsin marble. According to this figure, the GMTS criterion provides very good estimates for the SCB test results. As mentioned earlier, the mixed mode I/II fracture toughness data of marble obtained from fracture tests on the SCB specimens were generally lower than the estimates by common fracture criteria like the MTS criterion (see Fig. 3). In particular, for predominantly mode II conditions, there is a significant discrepancy between the experimentally obtained data and the theoretical results of the conventional criteria developed based on the singular stress term alone. Then the GMTS criterion was used and better estimates of the experimental results were provided. According to the GMTS criterion, a positive T -stress decreases mixed mode fracture resistance of a cracked body. Meanwhile, based on Fig. 6, the sign of T -stress for the tested SCB samples are generally positive except for a few cases in mode

I dominated loading conditions. Therefore, the reduction in the fracture toughness of the SCB specimen can be attributed to the effects of positive T -stress when the SCB specimen is subjected to mixed mode I/II.

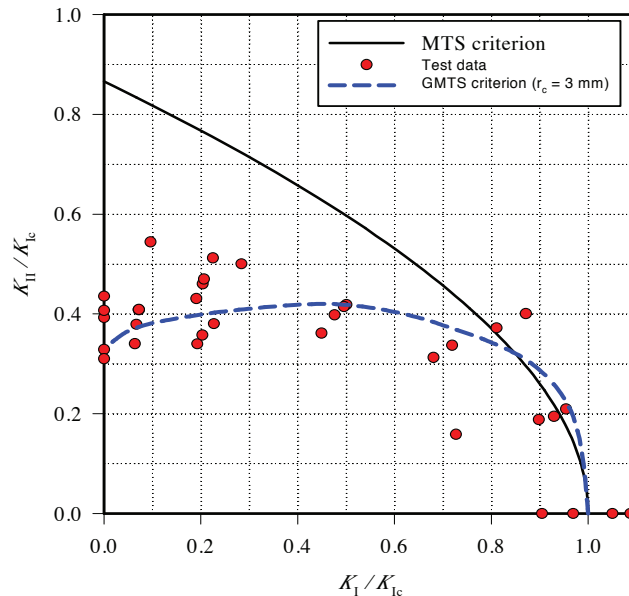


Fig. 6. Fracture curve of the GMTS criterion for mixed mode fracture resistance of tested SCB specimens made of Harsin marble

A review of recent pure mode II fracture toughness studies conducted on different rock samples using various test configurations [e.g. 12-14] suggests that the pure mode II fracture toughness (K_{IIc}) of rock materials is always significantly higher than the corresponding pure mode I fracture toughness (K_{Ic}). Such experimental findings sometimes have led the researchers working in the field of rock fracture mechanics to assume that $K_{IIc} > K_{Ic}$ is among the intrinsic properties of rock materials. However, according to the experimental results obtained in this paper for a marble rock, the average fracture toughness ratio K_{IIc}/K_{Ic} was about 0.4. In other words, the value of mode II fracture toughness in the tested SCB specimen was noticeably less than the K_{Ic} value. Therefore, one can conclude that the mode II fracture toughness K_{IIc} (or the fracture toughness ratio K_{IIc}/K_{Ic}) is very much dependent on the geometry and loading conditions in cracked specimens failing by brittle fracture. Indeed, those mode II crack specimens having a positive T -stress (like SCB specimen), exhibit a fracture toughness ratio K_{IIc}/K_{Ic} lower than 0.87; i.e. the figure suggested by the conventional MTS criterion with $T = 0$. The estimates of the GMTS criterion for pure mode II fracture toughness of the tested SCB specimen was also consistent with the experimentally obtained results.

6. Conclusions

- Conventional mixed mode I/II fracture criteria (based only on the stress intensity factors; K_I and K_{II}) such as the MTS criterion overestimated the mixed mode fracture toughness of a marble rock tested with SCB samples.
- A GMTS criterion was used for estimating the mixed mode I/II fracture toughness data. In the GMTS criterion, the effect of non-singular stress term (T -stress) was considered in addition to the stress intensity factors.
- Based on the GMTS criterion, reduction in the fracture resistance of the tested rock was mainly due to the effect of noticeable positive T -stress that exists in the SCB specimen especially for mode II dominated loading conditions.
- It was also shown that the GMTS criterion which uses three fracture parameters i.e. K_I , K_{II} and T can provide significantly better estimates for mixed mode fracture in the Harsin marble when tested by the SCB specimens.

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